NFSI & ICBEM, Banff, Canada, May 13-15, 2011

Adaptive Spatial Harmonic Analysis of EEG Data using Laplacian Eigenspace

Uwe Graichen*, Roland Eichardt*, Patrique Fiedler*, Daniel Strohmeier* and Jens Haueisen*
*Institute of Biomedical Engineering and Informatics
Faculty of Computer Science and Automation
Ilmenau University of Technology
Gustav-Kirchhoff-Str. 2, 98693 Ilmenau, Germany
Email: uwe.graichen@tu-ilmenau.de

***Abstract*—Electroencephalography is an important diagnostic tool for functional investigations of the human brain. Recent EEG measurement technologies provide high numbers of electrodes and sampling rates, which results in a considerable quantity of data. For the analysis of this EEG data, efficient signal analysis and decomposition methods are essential.**

In this paper a new method for spatial harmonic analysis of EEG data using the Laplacian eigenspace of the meshed surface of electrode positions is presented. The resulting eigenspace enables the spatial harmonic analysis, filtering, denoising and decomposition of EEG data.

For a proof of concept, the proposed approach is applied to an 128 channel EEG recording of visual evoked potentials. A set of harmonic spatial basis functions for the EEG electrode setup is estimated. The EEG data are spatially decomposed and low pass filtered using the harmonic spatial basis functions.

I. Introduction

Electroencephalography (EEG) is an important diagnostic tool to investigate the function of the human brain. During the last decades, EEG was established in many scientific and clinical applications, e. g. in neurology, cognitive neurosciences, and psychology.

Recent EEG measurement technologies provide up to 512 recording channels at sample rates of up to 20 kHz. This results in a considerable quantity of data, particularly for long term measurements. To analyze the recorded EEG data, efficient signal analysis and decomposition methods are essential.

Several methodologies that are applicable for a spatial decomposition of multichannel EEG data have been proposed previously. Classical examples are principal component analysis (PCA) [1], independent component analysis (ICA) [2] and parallel factor analysis (PARAFAC) [3]. Newer approaches are based, for example, on matching pursuit and utilize Bessel [4] or multichannel Gabor [5] atoms. Common application of Laplace spectra, which we use for the spatial decomposition, can be found in the field of graph theory [6] and computer graphics [7], [8]. Surface Laplacian approaches are applied in EEG to improve the spatial resolution [9]–[11]. Laplacian eigenmaps are also deployed for EEG studies [12], [13], in which EEG time series and topographies are investigated. However, the Laplacian eigenspace, used in these two approaches, does not consider the spatial arrangement of the EEG electrodes in $\mathbb{R}^3$.

In this paper we present a new method for spatial harmonic analysis of EEG data using the Laplacian eigenspace of the meshed surface of electrode positions. With the proposed approach we obtain basis functions of spatial harmonics for arbitrary arrangements of EEG electrodes. To decompose the recorded data, we simply multiply the data with this harmonic basis. We show in one example the spatial decomposition using an 128 channel EEG recording of visual evoked potentials. In addition to a spatial harmonic analysis, this approach facilitates the rejection of noisy and erroneous components, as well as the compression of data.

II. Methods

A. Eigenspaces of the Continuous Laplacian Operator

We consider a compact Riemannian manifold (M, g), where M is a connected manifold, which is real differentiable C^∞. The function g defines for each point $p \in M$ the inner product of the tangent space.

If the manifold M has a boundary $B = \partial M$, it is assumed that M is oriented, and C^∞ also applies to the boundary B. The outward unit normal vector field on B is denoted by n.

For a real-valued function f with $f \in L^2(M, g)$ and f is $C^k, k \geq 2$, the Laplacian Δ is defined by

$$\Delta f = \mathrm{div}(\mathrm{grad}\, f)\,, \tag{1}$$

with the divergence div and the gradient grad.

If the manifold M possesses a boundary B several boundary conditions (BC) can be applied

$$f = 0, \text{ on } \partial M \qquad \text{Dirichlet BC} \tag{2}$$

$$\frac{\partial f}{\partial n} = n \cdot \nabla f = 0, \text{ on } \partial M \qquad \text{von Neumann BC} \tag{3}$$

$$\frac{\partial f}{\partial n} + af = 0, \text{ on } \partial M \qquad \text{Robin (mixed) BC} \tag{4}$$

with the del operator ∇.

Using the Laplacian, by solving the Laplacian eigenvalue problem

$$\Delta \phi = \lambda \phi\,, \tag{5}$$

a basis for a harmonic analysis of the Riemannian manifold can be determined by finding all eigenvalues $\lambda \in \mathbb{R}$ with an associated nontrivial eigenfunction ϕ, with $\phi \in C^2(M)$. The set of all eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$ defines the spectrum of M

$$\text{spec}(M) = \{\lambda_n\}_{n=1}^{\infty} = \{0 \leq \lambda_1 < \lambda_2 < \ldots < \lambda_\infty\}\,, \quad (6)$$

with $\lim_{n\to\infty} \lambda_n \to \infty$. The eigenspace of the eigenfunctions $\{\phi_n\}_{n=1}^{\infty}$ forms an orthonormal basis. The set of eigenfunctions $\{\phi_n\}_{n=1}^{\infty}$ can be used for spectral analysis of the function defined on the manifold M. The Laplacian eigenspace can be considered as a base of a generalized Fourier analysis [14]–[16].

B. Eigenspaces of the Discrete Laplacian Operator

In practical applications surfaces are often represented by meshes. In this paper triangulated meshes $M = \{V, E, F\}$ are assumed, with sets of vertices V, edges E and faces F. For each vertex $v_i \in V$ a neighborhood $i^\star$ can be defined by $i^\star = \{j : e(i,j) \in E\}$. The number of neighbors of a vertex v_i is given by $d_i = |i^\star|$.

For the edges of the mesh a weight function $w : V \times V \to \mathbb{R}$ can be defined. Several weighting functions can be used, e. g. $w(i,j) = ||v_i, v_j||^{-1}$ where $||.||$ is the Euclidean distance between the vertices v_i and v_j [17] and $w(i,j) = \cot(\alpha_{ij}) + \cot(\beta_{ij})$, with α_{ij} and β_{ij} are the two angles opposite of edge $e(i,j)$ [7], [18], [19]. Using the weight function $w(.)$, the degree of vertex v_i can be defined by

$$d(i) = \sum_{j \in i^\star} w(i,j)\,. \quad (7)$$

The discrete Laplacian operator Δ for mesh data is defined by the weighted average over the neighborhood of the vertices v_i

$$\Delta x_i = \sum_{j \in i^\star} w(i,j)(x_j - x_i)\,. \quad (8)$$

In matrix notation the Laplacian operator Δ can be written as

$$\Delta \vec{x} = -L\vec{x}\,, \quad (9)$$

with the Laplacian matrix

$$L_{ij} = \begin{cases} d(i) & \text{if} \quad i = j \\ -w(i,j) & \text{if} \quad (i,j) \in E \\ 0 & \text{otherwise}\,. \end{cases} \quad (10)$$

The eigenvectors $\vec{x}$ and eigenvalues λ of L are computed

$$L\vec{x} = \lambda\vec{x}\,. \quad (11)$$

Since the Laplacian matrix L is real and symmetric, all eigenvalues are $\lambda \in \mathbb{R}$ and $\lambda \geq 0$. The eigenvectors $\vec{x}$ form an harmonic orthonormal basis. The corresponding eigenvalues λ can be considered as frequencies. The eigenvectors $\vec{x}$ can be used for a spectral analysis of functions defined on the mesh M.

The Laplacian matrix L is a square matrix $n \times n$, where n is the number of vertices. Furthermore, it is a sparse and symmetric matrix. The eigenvalues and eigenvectors of L are computed using a parallel implementation of the LAPACK routine DSYEVR [20]. DSYEVR consists of the two subroutines DSYTRD and DSTEMR. The DSYTRD-routine performs a Householder tridiagonalization of L and has a complexity of $O(n^3)$ [21]. The second subroutine DSTEMR computes the eigenvalues and eigenvectors of the resulting real symmetric tridiagonal matrix and has a complexity of $O(n^2)$ [22]. Thus the computational complexity of DSYEVR is $O(n^3)$.

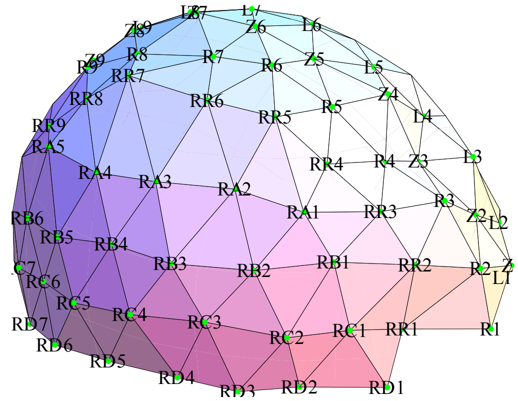

Fig. 1. The digitized electrode position of the EEG montage.

III. DATA

We applied the proposed method to a data set from a previously performed EEG experiment addressing cortical activation related to pattern reversal visual evoked potentials (VEP). The experiment employed a checkerboard visual stimulation according to ISCEV standards [23]. For proof of concept of the introduced approach, the EEG electrode positions, see figure 1, and the recorded time series were analyzed. The averaged and filtered VEP data are shown in figure 2. The position of the EEG electrodes was tracked using an optical Advanced Neuro Technology xensor 3D electrode digitizer system. The experiment was applied to a single subject (male, age 23). During the experiment, the subject focused on the center of a periodically changing checkerboard pattern with an inter stimulus interval of 550 ms. EEG signals were recorded simultaneously using an Advanced Neuro Technology WaveGuard 128-channel EEG

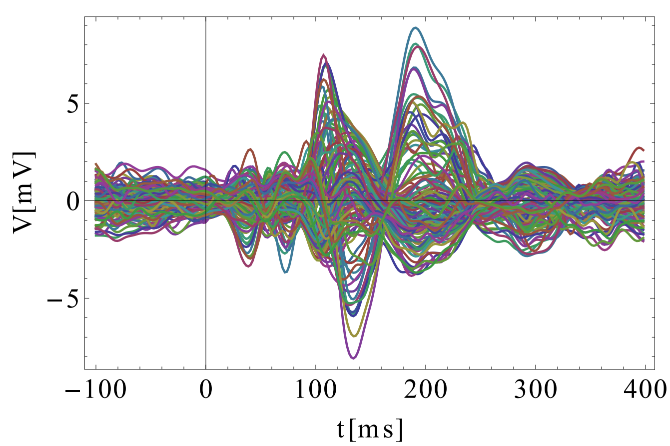

Fig. 2. The VEP recorded during the experiment employing a checkerboard pattern reversal visual stimulation

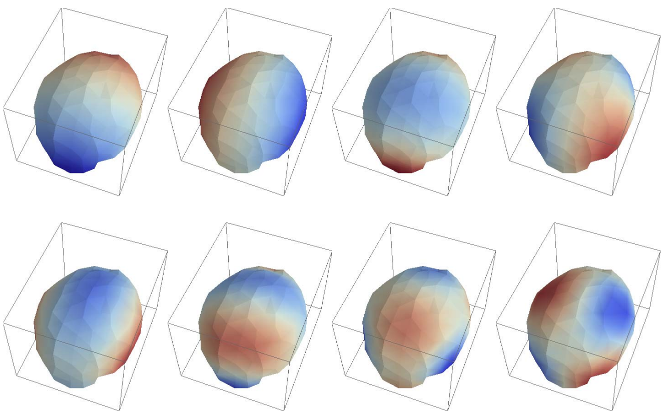

Fig. 3. Illustration of the 2nd to the 9th low frequency base functions, from upper left to lower right

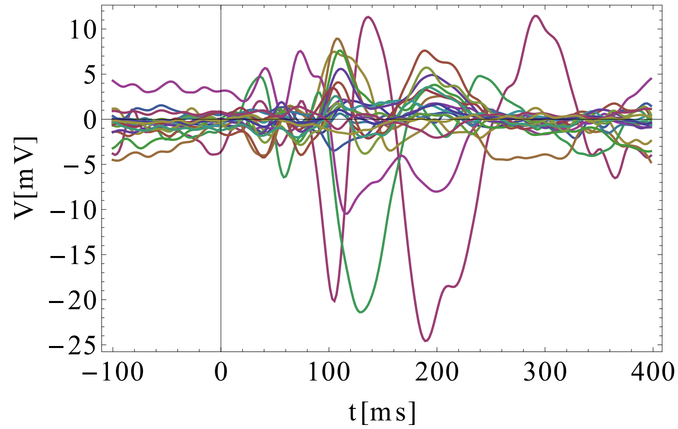

Fig. 5. The harmonic spatial decomposition of the VEP data deploying the 20 basis functions (eigenvectors) with the lowest frequencies

headcap with an equidistant electrode layout and an asa-lab 128 channel amplifier with common average reference. Data were sampled at 512 Hz and software filtered using a passband 24 dB/oct from 0.5 to 40 Hz. 150 trials were averaged.

IV. Results

In the first step the weighted Laplacian matrix L was created using information about the topology of EEG electrode montage and the measured electrode positions. To generate the Laplacian matrix, the edges $e(i, j)$ between the vertices v_i and v_j of the triangular mesh were weighted by $w(i, j) = ||v_i, v_j||^{-1}$, with the Euclidean distance $||.||$. This weight function was chosen to decrease the influence on the Laplacian of more distant vertices. The weighted Laplacian matrix is a square matrix $L \in \mathbb{R}^{n \times n}$, where n is the number of electrodes. The eigenspace decomposition of L was computed, the resulting eigenvectors form a set of harmonic spatial basis functions, which can be applied for the spatial decomposition of EEG data. The EEG data were decomposed by the multiplication of the matrices XD. The row vectors of the matrix $X \in \mathbb{R}^{n \times n}$ are the eigenvectors $\vec{x}$ of L, and $D \in \mathbb{R}^{n \times m}$ contains the measured data, with the number of EEG channels n and the number of time samples m. In figure 3 the second to the ninth low frequency basis functions are presented (the DC basis function is not displayed).

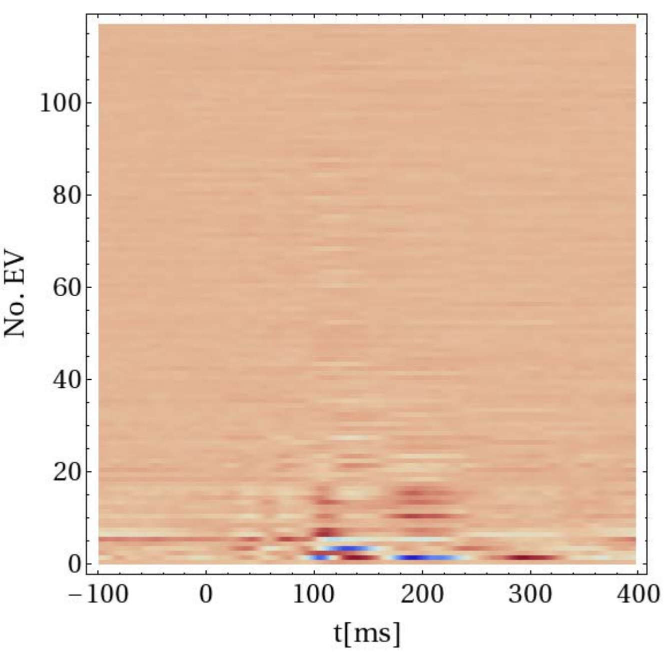

Fig. 4. The time spatial-frequency representation of the VEP data, the basis functions (eigenvectors) used for the decomposition are sorted by frequency from the bottom to the top of the image, starting with the low frequency at the bottom, the numbers of the basis functions are shown at y-axis

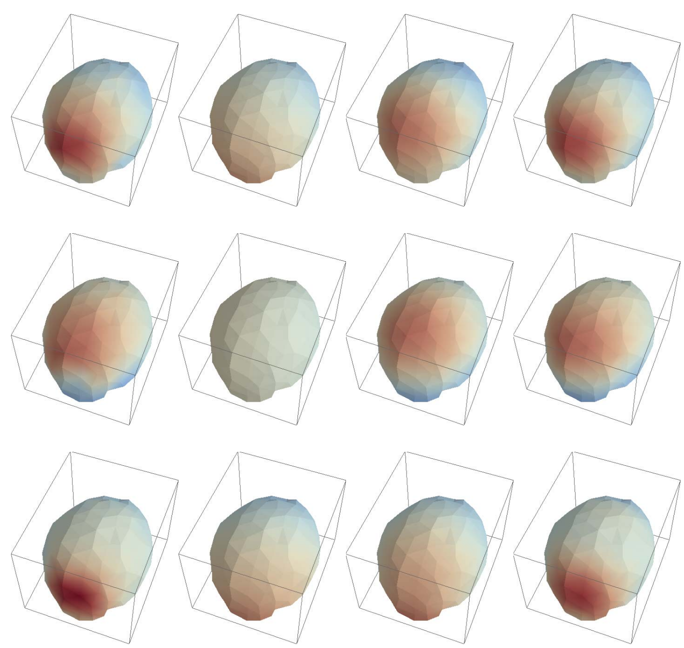

Fig. 6. Reconstruction of the spatial current distribution at different time steps after stimulation (first to third row, 105 ms, 118 ms and 190 ms respectively) using a limited number of low frequency basis functions (first column to fourth column, original data, 3, 10 and 20 basis functions, respectively)

The contribution of spatial basis functions at each time to the VEP is shown in figure 4, which is a time-spatial frequency representation. The main contribution to the VEP is provided by the low frequency basis functions, shown at the bottom of figure 4 and in figure 5.

The basis functions can also be used for a spatial filtering and noise suppression of the EEG data. For a spatial low pass filtering only a small number of low frequency basis functions are used to reconstruct the spatial electrical activity distribution on the scalp. In figure 6 each row of images represents a different time position of the VEP: 106 ms upper row, 113 ms middle row and 149 ms. The unfiltered data are shown in the 1st column. The filtered data using 3, 10 or 20 basis functions for reconstruction are pictured in the columns 2 – 4.

V. Discussion and Conclusion

In this paper a new harmonic spatial analysis method for EEG data is introduced. The eigenvectors of the discrete mesh Laplacian matrix form a harmonic orthonormal base, which can be adaptively computed using the topology of the EEG montage and the electrode positions in $\mathbb{R}^3$. The set of basis functions enables the harmonic spatial decomposition of EEG data. The proposed method can be deployed to arbitrary electrode setups and furthermore to other sensor arrays like MEG.

The eigenvectors of the discrete mesh Laplacian matrix needs to be calculated only once for an electrode setup if standard electrode positions are used, or once for each measurement if the electrode positions were tracked individually. Thereafter the decomposition of the recorded time series can be done by a multiplication of two matrices containing the eigenvectors and the time series.

At a standard desktop computer with a 3.00 GHz Intel Core2 Duo CPU E6850 and 8 GByte RAM, the generation of the Laplacian matrix and the computing of the eigenvalues and eigenvectors takes 0.0296 ± 0.0018 s. The decomposition of the recorded VEP data with 256 time samples and 128 channels consumes 0.0033 ± 0.0003 s. In both cases 1000 runs are averaged to estimate the computing time.

We have applied the method to an EEG electrode setup and we analyzed VEP time series related to pattern reversal visual stimulation. As shown, the main contribution to the recorded VEP data is provided by low frequency spatial basis functions. Additionally, the introduced approach is applicable for a spatial low pass filtering and denoising of the data.

The proposed approach can be easily combined with frequency and time-frequency analysis methods.

Acknowledgment

This work was supported by the German Federal Ministry of Education and Research (03IP605).

References

[1] T. D. Lagerlund, F. W. Sharbrough, and N. E. Busacker, "Spatial filtering of multichannel electroencephalographic recordings through principal component analysis by singular value decomposition," *Journal of Clinical Neurophysiology*, vol. 14, no. 1, pp. 73–82, 1997.

[2] T. P. Jung, S. Makeig, M. Westerfield, J. Townsend, E. Courchesne, and T. J. Sejnowski, "Analysis and visualization of single-trial event-related potentials," *Human Brain Mapping*, vol. 14, no. 3, pp. 166–185, 2001.

[3] F. Miwakeichi, E. Martinez-Montes, P. A. Valdes-Sosa, N. Nishiyama, H. Mizuhara, and Y. Yamaguchia, "Decomposing EEG data into space-time-frequency components using parallel factor analysis," *Neuroimage*, vol. 22, no. 3, pp. 1035–1045, 2004.

[4] M. Gratkowski, J. Haueisen, L. Arendt-Nielsen, A. C. N. Chen, and F. Zanow, "Decomposition of biomedical signals in spatial and time-frequency modes," *Methods of Information in Medicine*, vol. 47, no. 1, pp. 26–37, 2008.

[5] M. Gratkowski, J. Haueisen, L. Arendt-Nielsen, and F. Zanow, "Topographic matching pursuit of spatio-temporal bioelectromagnetic data," *Przeglad Elektrotechniczny*, vol. 83, no. 11, pp. 138–141, 2007.

[6] F. R. K. Chung, *Spectral Graph Theory*. American Mathematical Society, 1997, vol. 92, cBMS Regional Conference Series in Mathematics.

[7] G. Taubin, "Geometric signal processing on polygonal meshes," in *STAR Proceedings of Eurographics 2000*, S. Coquillart and D. Duke, Eds. Eurographics Association, 2000.

[8] H. Zhang, O. van Kaick, and R. Dyer, "Spectral methods for mesh processing and analysis," in *STAR Proceedings of Eurographics*, D. Schmalstieg and J. Bittner, Eds., vol. 92, no. RC-20404, 2007, pp. 1–22.

[9] L. A. Bradshaw and J. P. Wikswo, "Spatial filter approach for evaluation of the surface laplacian of the electroencephalogram and magnetoencephalogram," *Annals of Biomedical Engineering*, vol. 29, no. 3, pp. 202–213, 2001.

[10] P. L. Nunez and A. F. Westdorp, "The surface laplacian, high resolution EEG and controversies," *Brain Topography*, vol. 6, no. 3, pp. 221–226, 1994.

[11] P. L. Nunez, R. B. Silberstein, P. J. Cadusch, R. S. Wijesinghe, A. F. Westdorp, and R. Srinivasan, "A theoretical and experimental study of high resolution EEG based on surface laplacians and cortical imaging," *Electroencephalography and Clinical Neurophysiology*, vol. 90, no. 1, pp. 40–57, 1994.

[12] A. Gramfort and M. Clerc, "Low dimensional representations of MEG/EEG data using laplacian eigenmaps," in *Joint Meeting of the 6th International Symposium on Noninvasive Functional Source Imaging of the Brain and Heart and the International Conference on Functional Biomedical Imaging*, 2007, pp. 169–172.

[13] M. Ramirez-Velez, R. Staba, D. S. Barth, and F. G. Meyer, "Nonlinear classification of EEG data for seizure detection," in *3rd IEEE International Symposium on Biomedical Imaging*, 2006, pp. 956–959.

[14] I. Chavel, *Eigenvalues in Riemannian Geometry*, ser. Pure and Applied Mathematics. Academic Press, 1984, vol. 115.

[15] S. Rosenberg, *The Laplacian on a Riemannian Manifold: An Introduction to Analysis on Manifolds*, ser. London Mathematical Society Student Texts. Cambridge University Press, 1997, no. 31.

[16] M. Berger, *A Panoramic View of Riemannian Geometry*. Springer, 2003.

[17] G. Taubin, "Signal processing approach to fair surface design," in *Proceedings of the ACM SIGGRAPH Conference on Computer Graphics*, 1995, pp. 351–358.

[18] M. Desbrun, M. Meyer, P. Schroder, and A. H. Barr, "Implicit fairing of irregular meshes using diffusion and curvature flow," in *Proceedings of SIGGRAPH 99: 26th International Conference on Computer Graphics and Interactive Techniques*, 1999, pp. 317–24.

[19] O. Sorkine, "Differential representations for mesh processing," *Computer Graphics Forum*, vol. 25, no. 4, pp. 789–807, 2006.

[20] E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen, *LAPACK Users' Guide*, 3rd ed. Society for Industrial and Applied Mathematics, 1999.

[21] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Johns Hopkins University Press, 1996.

[22] I. S. Dhillon and B. N. Parlett, "Multiple representations to compute orthogonal eigenvectors of symmetric tridiagonal matrices," *Linear Algebra and its Applications*, vol. 387, pp. 1–28, 2004.

[23] J. V. Odom, M. Bach, M. Brigell, G. E. Holder, D. L. McCulloch, A. P. Tormene, and Vaegan, "ISCEV standard for clinical visual evoked potentials (2009 update)," *Documenta Ophthalmologica*, vol. 120, no. 1, Sp. Iss. SI, pp. 111–119, 2010.