Abstract: We have applied the reciprocity theorem to the calculation of the lead field of EEG leads. A realistic head model based on MR images was constructed and the finite difference method was applied in obtaining the forward solutions. The lead field information can be used for the source localization in the inverse procedure. The strengths of the lead field approach are that the forward solution is calculated only once, the lead field coefficients are calculated for each node of the model, and thus, each node is a possible source node in the inverse solution.

INTRODUCTION

The general theory of reciprocity was introduced by Hermann von Helmholtz in 1853. Reciprocity is a common property for all linear systems. The theory can be formulated using a quasi-static approach as follows: As the current flows between two surface electrodes through a volume conductor, the current distribution expresses how the same electrodes record (i.e., how sensitive they are to) potentials caused by dipole sources at any place within the volume conductor. The concept of the lead field was first introduced by McFee and Johnston in 1953 [1].

The reciprocity theorem has been applied to EEG studies by Rush and Driscoll [2]. They studied the sensitivity of EEG leads to the location and orientation of sources in the brain using a three-concentric-sphere model of the head. A more recent study by Fletcher et al. presents a lead field analysis algorithm for the calculation of the surface potentials based on the boundary element method (BEM) and a realistic volume conductor model [3].

Our purpose was to calculate the EEG lead fields for the inverse calculations by using a numerical model based on individual geometry of the subject. The calculations were carried out by using the finite difference method (FDM) originally developed in our institute for thorax modelling [4].

METHODS

The lead field can be determined by feeding to the lead a reciprocal current \( I = 1 \text{A} \), which generates an electric potential field \( \Phi_{I} \) in the volume conductor. The reciprocal current field, which is also defined as the lead field, can be written:

\[
\vec{J}_{RS} = \sigma \vec{E}_{RS}
\]  

(1)

where \( \vec{E}_{RS} \) is the negative gradient of the electric potential field and \( \sigma \) is the conductivity of the medium. If the lead field \( \vec{J}_{RS} \) is known the voltage \( V_{AB} \) in the lead due to current sources \( I \)

\[
V_{AB} = \int_{A}^{B} \vec{J}_{RS} \cdot \vec{r} \, dv
\]  

(2)

within the volume conductor can be calculated (refer also Figure 1) [5]:

The anatomical data for the head model was obtained from T1-weighted MR images (GE Vectra 0.5T) of a neurological female patient. One hundred transversal, gradient-echo images of 2mm thickness with 1.3 x 1.3 mm\(^2\) resolution in plane were used. An image segmentation procedure [6] was applied and a three-dimensional head model consisting of 470 000 elements was constructed. The applied resistivity values for different tissues are given in Table I, adapted from [2].

The international 10-20 electrode system was used, and the electrode positions were marked with capsules filled with oil for MR imaging. The electrode positions were localized manually from the MR images and stored in the same coordinate system as the head model.

![Figure 1. The setup for the reciprocal calculation.](image)

Table I  Applied resistivity values

<table>
<thead>
<tr>
<th>Tissue</th>
<th>[(\Omega\text{cm})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalp</td>
<td>222</td>
</tr>
<tr>
<td>Skull</td>
<td>17760</td>
</tr>
<tr>
<td>CSF</td>
<td>222</td>
</tr>
<tr>
<td>White matter</td>
<td>222</td>
</tr>
<tr>
<td>Gray matter</td>
<td>222</td>
</tr>
</tbody>
</table>

The lead field of each EEG channel was obtained by feeding reciprocal current to the electrodes and calculating the resulting sensitivity distribution. This corresponds to the
calculation of the forward problem using the electrodes as sources. In our study we used bipolar measurements so that Cz was used as a second recording electrode in all recordings. The sources were set up by selecting 36 nodes closest to the corresponding electrode location on the scalp (electrode size approximately 1.5 cm²). The reciprocal current generator was defined by applying ±100 V to the source nodes and defining a 10 kΩ resistance between the source nodes and the neighbouring nodes. The potential distribution throughout the model was calculated iteratively using the finite difference method [7,8] and the termination accuracy was 10⁻⁷ as a change in the absolute value. The total current through electrodes was scaled to 1 A before applying Eq. 1 in order to equalize the differences between different electrode pairs.

The computations were done on DEC 3000 AXP Alpha cluster computers (225–266 MHz), running the Digital UNIX V3.2C operating system.

RESULTS

The forward solution was calculated for 20 electrode pairs and the corresponding lead field coefficients for every 470 000 nodes were obtained. As an example, Figure 2 shows the lead field distribution of the lead F7–Cz in the brain. According to the Equation 1, the lead field coefficients can be used to calculate the potential at all the electrode locations due to a dipole source anywhere within the volume conductor model. The number of iterations required to achieve the termination rule was from 47000 to 81000.

DISCUSSION

The lead field coefficients of the international 10-20 system EEG leads were determined. This was done by reciprocal energization of the lead pairs using a realistic volume conductor model of the head. The set of linear equations were solved using the finite difference method.

It should be noted that even if the resistivity values of the white matter, gray matter and cerebro spinal fluid (CSF) differed it would not increase the complexity of the model compared to the situation where all these tissues were considered as one entity.

The method is quite computer intensive, but the strength of the lead field approach is based on the fact that the forward problem (i.e. the lead field for each electrode pair) has to be solved only once in order to find the inverse solution.

The forward solution can be used in Equation (2) in determining the optimal source localization based on minimizing the difference between the measured and calculated scalp potentials.

Further, the lead field approach allows a wide range of source distributions studied using the same lead field data. For example, the lead field coefficients can be used for analyzing different evoked potentials in the source localization procedure. The source(s) can be anywhere within the volume conductor model, the solution is not restricted, for example, on the tissue boundaries.

REFERENCES


Figure 2. The lead field distribution of the lead F7-Cz.